Edge Verifiability: Characterizing Outlier Measurements for Wireless Sensor Network Localization

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Outline

- Background
- Edge verifiability
- Graph verifiability
- Evaluation
- Discussion and Conclusion

Background*

- A large number of WSN localization approaches are presented.
  - High accuracy
  - High efficiency
  - Low cost
- Why do these well-designed approaches always have unpleasant performance in practical uses?
  - Ranging errors

Background*

- Ranging Errors
  - Outliers
    - Large errors, due to hardware malfunction or failure…
    - Destroy localization
    - Must be eliminated: detect and remove outliers
  - Noises
    - Small errors, due to environmental factors, systematic errors, …
    - Degrade the location accuracy
    - Need to be alleviated: minimization non-consistency (e.g., multilateration), reducing weights of noisy data (e.g., SISR)

Background*

- The straightforward method to deal with noisy and outlier ranging is triangle inequality,
- Triangle inequality has its own limitations that make itself far from accurate and reliable.
  - Coarse granularity
  - Identification

Goal

Detect outlier distance measurements for localization

Problem Definition

- Network Model
  - A network N can be modeled by a distance graph G = <V, E, W> where each vertex $v_i \in V$ denotes a node and each edge $e \in E$ indicates the neighborhood of its two endpoint nodes.
  - The measured distance between two neighboring nodes is defined by W(e) for $e = (v_i, v_j) \in E$.
- Give a distance graph model $G = <V, E, W>$ and edge $e = (u, v) \in E$
  - Edge verification: Validate correctness of $W(e)$.
  - Edge verifiability: Telling if edge e is verifiable.

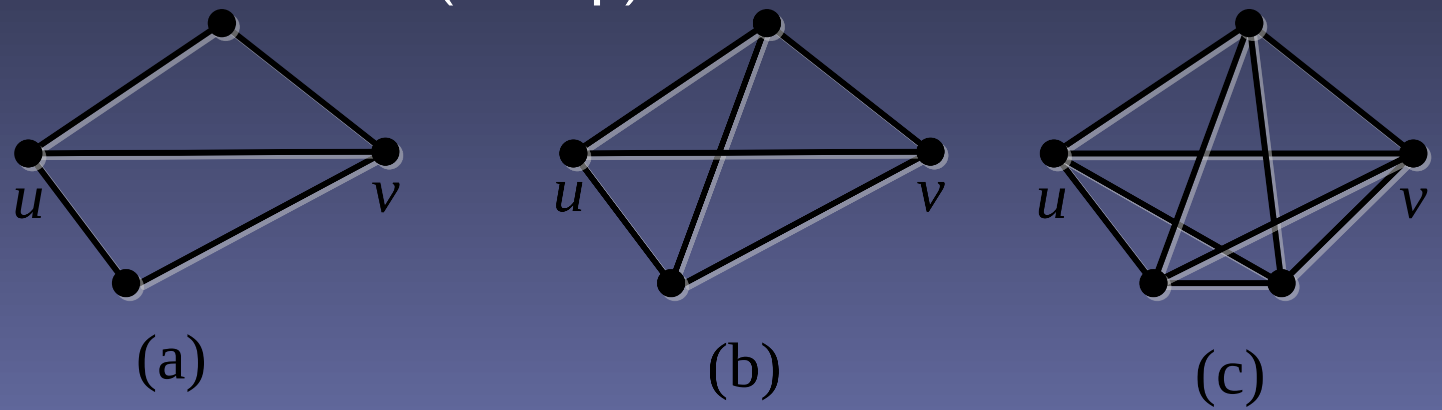
Approach

- Exploit the remaining graph *G-e* after the removal of *e*.
- For a (satisfied) realization (*G-e*, *p*) of *G-e*, the value of $||p(u)-p(v)||$ can be used to verify *e*.

Assume *G-e* provides enough correct information to verify *e*.

Approach

- G-e falls into 3 cases in terms of rigidity
  - If G-e is flexible, the value of ||p(u)-p(v)|| falls in a number of continuous intervals.
  - If G-e is rigid, ||p(u)-p(v)|| has a series of discrete values.
  - If G-e is globally rigid, ||p(u)-p(v)|| has a unique value for all (G-e, p).

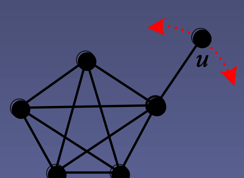


Approach

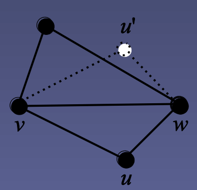
- The first two cases indicate whether e is reasonable; i.e., W(e) in a reasonable range.
  - 1st case: u and v are included in a cycle in G-e. "triangle inequality" is a special case.
  - 2nd case: "beyond triangle inequality" partially solves this case.
- The 3rd case guarantees the correctness (or incorrectness) of e.
  - **This work focuses on the 3rd case: G-e is globally rigid.**

Preliminaries

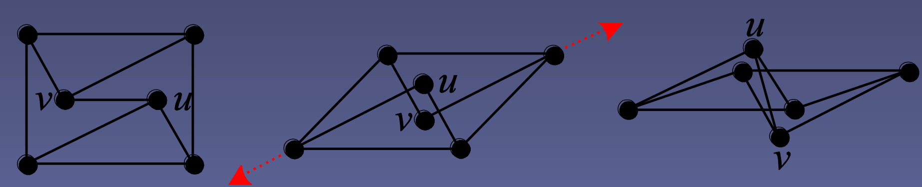
- Graph Rigidity and Realization
  - A realization of graph G is a pair (G, p) where p is a function mapping the vertices in V to points in a Euclidean space
  - Non-uniqueness of realization
- **Theorem 1** (Jackson at. el.). A graph with n>4 vertices is globally rigid in 2 dimensions if and only if it is 3-connected and redundantly rigid.



Not rigid.



Not 3-connected.



Not redundantly rigid.

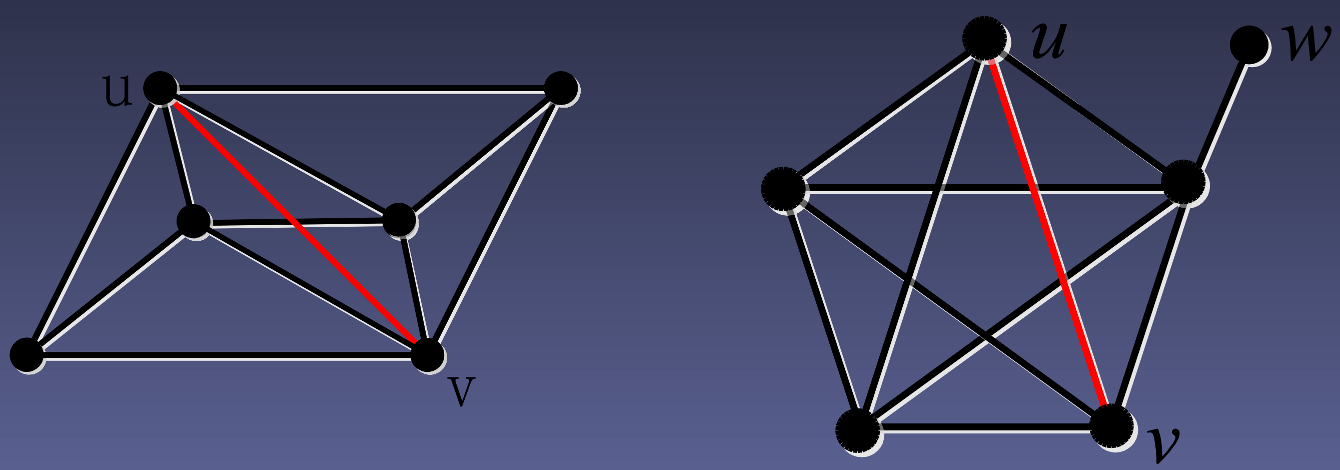
Edge Verifiability

- **Definition 1**. Given a distance graph G = <V, E, W>, an edge e = (u, v) $\in$ E is a normal edge if $W(e) = ||\pi(u) - \pi(v)||$; otherwise, e is an outlier edge.

- **Definition 2**. Given a distance graph G=<V, E, W>, an edge e = (u, v) $\in$ E is referred to be *verifiable* if $||p(u) - p(v)|| = ||q(u) - q(v)||$ holds for any two equivalent realizations (G-e, p) and (G-e, q) of G-e.

Edge Verifiability

- **Theorem 2**. In a graph G = <V, E, W>, an edge e = (u, v) $\in$ E is verifiable in G if G-e is globally rigid, or the two ends u and v are included in at least one globally rigid component of G-e.



Graph Verifiability

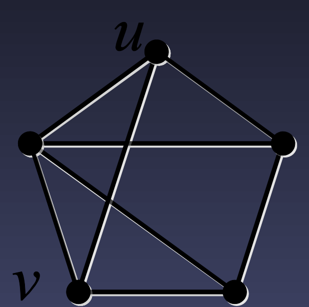
Expand the concept of verifiability from edges to graphs

- **Definition 3**. Given a distance graph G = <V, E, W> G is verifiable if e is verifiable for all e ∈ E.

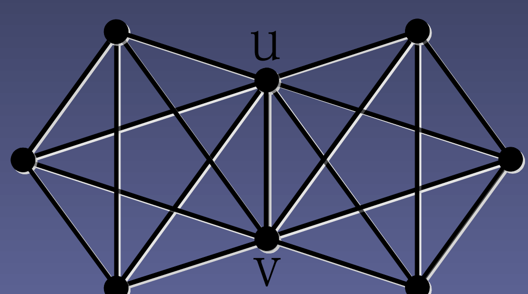
- **Theorem 3**. Given a globally rigid graph G = <V, E, W>, G is verifiable if and only if G-e is globally rigid for all e ∈ E.

Graph Verifiability

- **Theorem 4.** For a graph G = <V, E, W> on n > 5 vertices, if $\delta(G) \geq \frac{n+1}{2}$ where $\delta(G)$ denotes the minimum degree in G, G is verifiable.



$\delta(G) \geq \frac{n+1}{2}$, n=4



$\delta(G) \geq \frac{n}{2}$

Lemma 2. Let G = (V,E) be a graph on n ≥ 4 vertices with $\delta(G) \geq \frac{n+1}{2}$, where $\delta(G)$ denotes the minimum degree in G, G is globally rigid. [Jackson et al]

Graph Verifiability

- Easily verifiable graph
  - **Definition 4**. Graph G = <V, E, W> is easily verifiable if G-e is easily localizable for all e $\in$ E.
  - **Theorem 5**. If G is a quadrilateration graph, G is easily verifiable.

G-e is a trilateration, graph which is easily localization.

Discussion

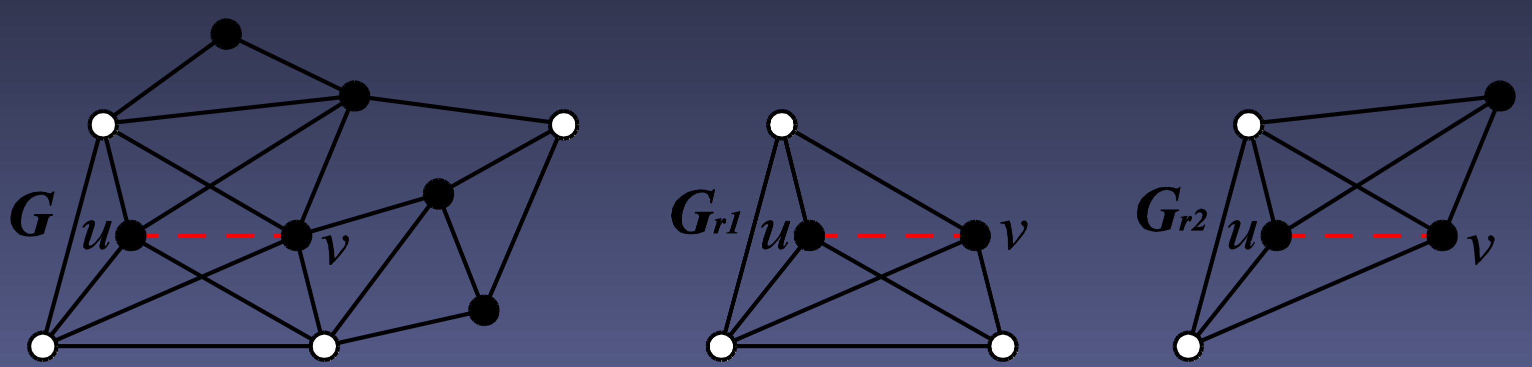
G-e may contain outliers, i.e., G-e may provide incorrect information!

Minimally globally rigid component(MGRC)

A graph G = (V,E) is minimally globally rigid if G is globally rigid, and no proper subgraph of G is globally rigid.

Discussion

- MGRC Rule
  - Verify each edge using one of its MGRCs.
  - Discovery: Find a MGRC
  - Nonuniqueness: MGRC may be not unique



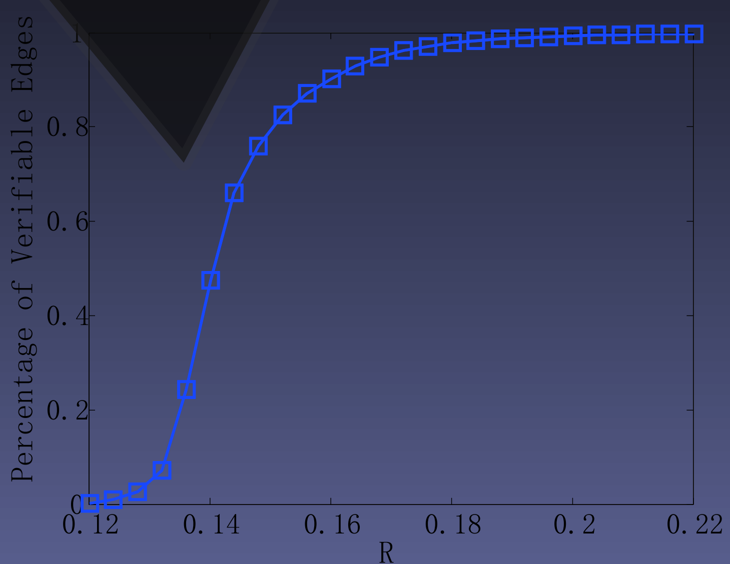
Evaluation

- We randomly generate networks of 100 nodes which are uniformly distributed in a unit square $[0, 1]^2$
- For each network instance, we change the communication range R from 0.12 to 0.22 stage-by-stage with a step length of 0.04.
- For each setting, we integrate results from 100 network instances.

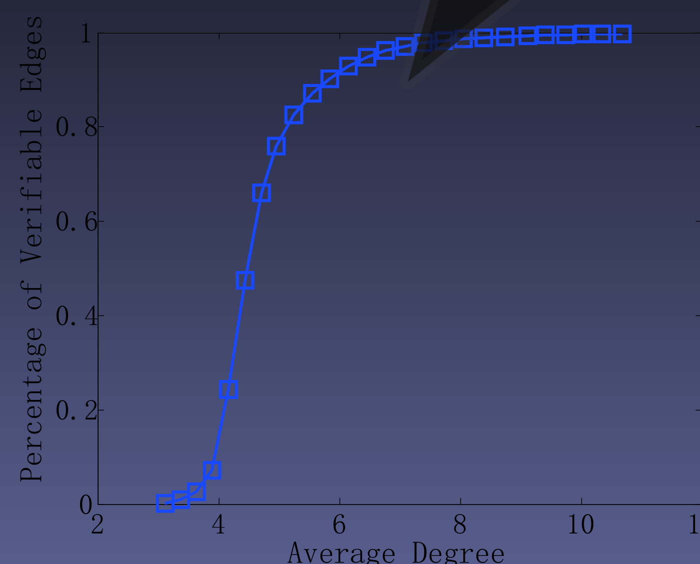
Evaluation

percentage of verifiable edges increases sharply to over 90% as average degree grows from 4 to 6.

more edges are verifiable in dense networks, since sufficient distance constraints are available.



Percentage of verifiable edges v.s. R

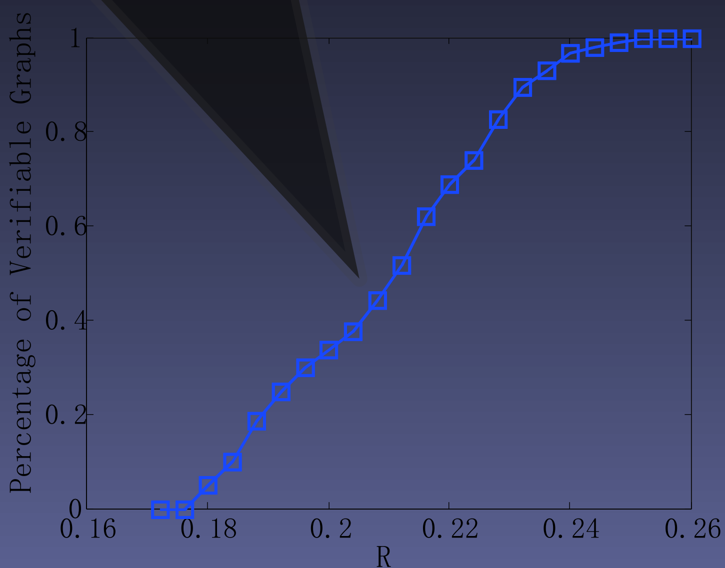


Percentage of verifiable edges v.s. Average Degree

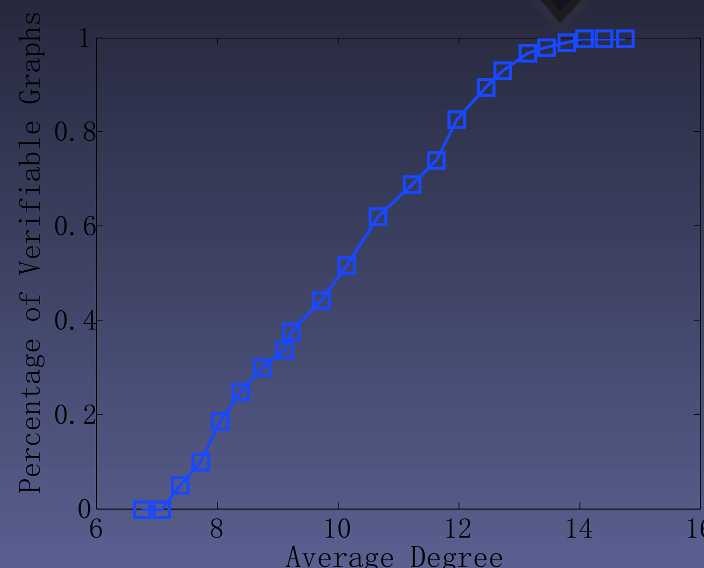
Evaluation

Percentage of verifiable graphs increases near-linearly as R grows from 0.18 to 0.24.

All graphs are verifiable when the average degree is over 14.



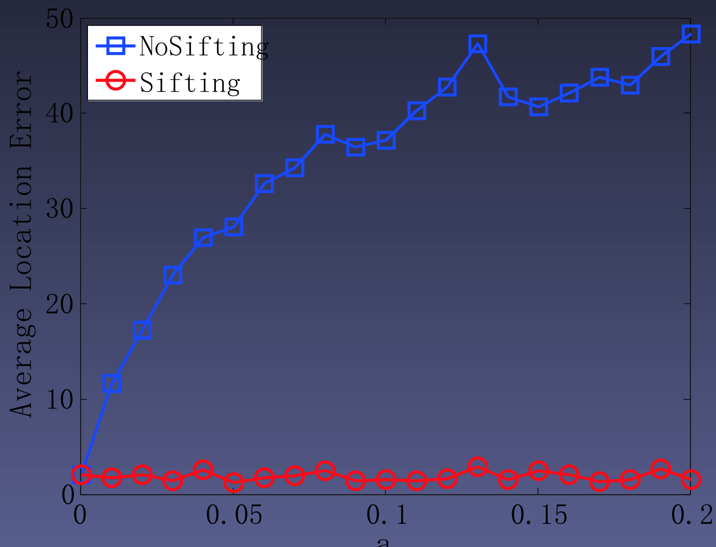
Percentage of verifiable graphs v.s. R



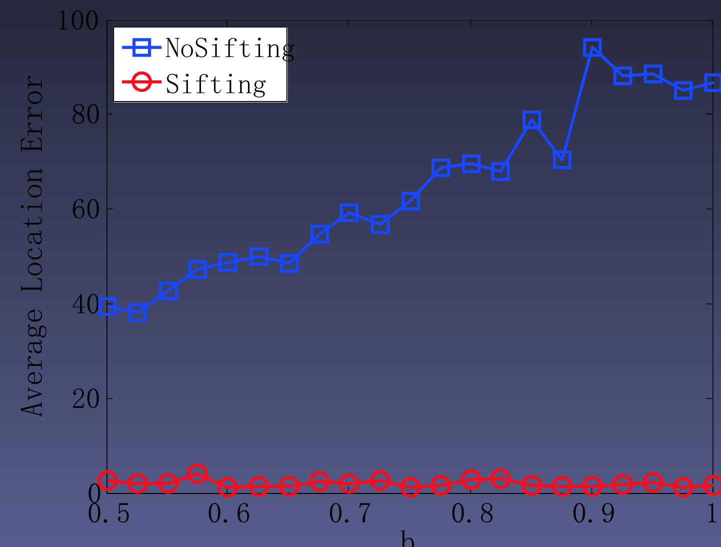
Percentage of verifiable graphs v.s. Average Degree

Evaluation

outlier sifting vs no sifting
in WSN localization



error of outliers b=0.5



percentage of outliers in the graph a = 0.1

Conclusions

- Explore how to identify outliers by using redundant distance information.
- Establish the theoretical foundation of outlier detection problem.
- Analyze the conditions of a specific measurement being an outlier.