Edge Verifiability: Characterizing Outlier Measurements for Wireless Sensor Network Localization

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***Abstract*—A majority of localization approaches of wireless sensor networks rely on the measurements of inter-node distance. Errors are inevitable in distance measurements and we observe that a small number of outliers can degrade localization accuracy drastically. To deal with noisy and outlier ranging results, a straight-forward method, triangle inequality, is often employed in previous studies. However, triangle inequality has its own limitations that make itself far from accurate and reliable. In this study, we first analyze how much information are needed to identify outlier measurements. Applying rigidity theory, we propose the concept of verifiable edges and derive the conditions of an edge being verifiable. On this basis, we design a localization approach with outlier detection, which explicitly eliminates the rangings with large errors before location computation. Considering entire networks, we define verifiable graphs in which all edges are verifiable. If a wireless network meets the requirements of graph verifiability, it is not only localizable, but also outlier-resistant. Extensive simulations are conducted to examine the effectiveness of the proposed approach. The results show the remarkable improvements of location accuracy by sifting outliers.**

I. Introduction

The proliferation of wireless and mobile devices has fostered the demand for context-aware applications, in which location is viewed as one of the most significant contexts. For wireless sensor networks, location information provides essential contexts for data interpretation and network operations.

In recent years, a number of techniques have been proposed for in-network localization, in which a portion of special nodes (called beacons or anchors) know their global locations and the rest determine their locations by measuring the Euclidean distances to their neighbors. Several distance ranging methods, such as Radio Signal Strength (RSS) [1] and Time Difference of Arrival (TDoA) [2], are adopted in practical systems. Based on these techniques, the ground truth of a wireless ad-hoc network can be modeled as a distance graph $G = \langle V, E, W \rangle$, where V is the set of wireless nodes, E is the set of links, and $W(u, v)$ is the distance measurement between a pair of nodes u and v. The problem of localization is to figure out the locations of other nodes based on inter-node distance measurements and global locations of those beacons.

In practice, however, noises and outliers are inevitable in distance ranging. The ranging errors generally come from the following three sources.

Hardware malfunction or failure. Distance measurements will be meaningless when encountering ranging hardware malfunction. Besides, incorrect hardware calibration and configuration also deteriorate ranging accuracy, which is not much emphasized by previous studies. For example, RSS suffers from transmitter, receiver, and antenna variability, and the inaccuracy of clock synchronization results in ranging errors for TDoA.

Environmental factors. RSS is sensitive to channel noise, interference, and reflection, all of which have significant impacts on signal amplitude. The irregularity of signal attenuation, especially in complex indoor environments, increases remarkably. In addition, the signal propagation speed often exhibits variability as a function of temperature and humidity for the propagation time based ranging measurements, so we cannot assume the propagation speed is constant across a large field.

Adversary attacks. As location-based services become more and more prevalent, the localization infrastructure is becoming the target of adversary attacks. By reporting fake location or ranging results, an attacker, e.g., a compromised (malicious) node, can completely distort the coordinate system. Different from the previous cases, the outliers here are intentionally generated by adversaries.

Ignoring the existence of outliers is not a choice to deal with them. To detect outliers, a straight-forward solution is to judge graph embeddability based on triangle inequality. A graph violating triangle inequality is by no means embeddable. Triangle inequality, however, has serious drawbacks in the following two aspects:

Coarse granularity. For a triangle △ABC, |AB|=5,

|AC|=5, and |BC|=5. Suppose for some reason, the distance between B and C is wrongly measured as 1, or 9; i.e., |BC|=1 or 9. For both two cases, △ABC is embeddable and triangle inequality fails to detect the outlier measurement |BC|. Triangle inequality is only a coarse-grained evidence of the inexistence of outliers and cannot guarantee the correctness of satisfied measurments.

Identification. Triangle inequality just indicates the existence of outliers, but cannot identify them. Formally, a triangle violating the inequality implies surely that at least one distance measurements are incorrect, but we have no idea which are them.

Such limitations motivate us to design a novel approach to detect outlier distance measurements beyond triangle inequality. We notice that a network can be uniquely located without using all inter-node distances. Thus for a graph G and a pair of neighboring vertices u and v, if $G-(u,v)$ contains enough correct information on the location u and v, the measured distance between u and v (denoted by $W(e)$ where $e=(u,v)$) is not necessarily used for localization, but can be further used for verification. That is to say, suppose we intentionally ignore $W(e)$, we still have some ideas to figure it out from $G-(u,v)$. Such redundant information in inter-node distances is a good starting point to characterize noises and outliers in distance measurements.

In this study, we analyze that how much information are needed to identify outlier measurements and how to use them. Applying rigidity theory, we propose the concept of verifiable edges and derive the conditions of an edge being verifiable. On this basis, we design a localization approach with outlier detection, which explicitly eliminates the rangings with large errors before location computation, thus increasing location accuracy. Considering entire networks, we define verifiable graphs in which all edges are verifiable. If a wireless network meets the requirements of verifiable graph, it is not only localizable, but also outlier-resistant. Extensive simulations are conducted to examine the effectiveness of the proposed approach. The results show the remarkably improvements of location accuracy by sifting outliers.

The rest of this paper is organized as follows. In Section II, we build the network model and investigate the theory of network localization. Section III presents the overview of edge verifiability. To identify outliers, we propose the concept of verifiable edges in Section IV. The conditions of an edge being verifiable are also studied in this section. In Section V, extending the results from a single edge to entire networks, we discuss verifiable graphs. Section VI presents the evaluation results from extensive simulations. We summarize the related work in Section VII, and conclude our work in Section VIII.

II. Preliminaries

A. Network Model

We suppose that each node is located at a distinct location in some region of a plane and associated with a specific set of "neighboring" nodes. Let $\mathbb{N}$ be a network of n nodes labeled $v_1, v_2, ..., v_n$. Let $\pi(v_i)$ denote the ground truth position of v_i. And we assume that a small portion of nodes, called *beacons*, are at known locations.

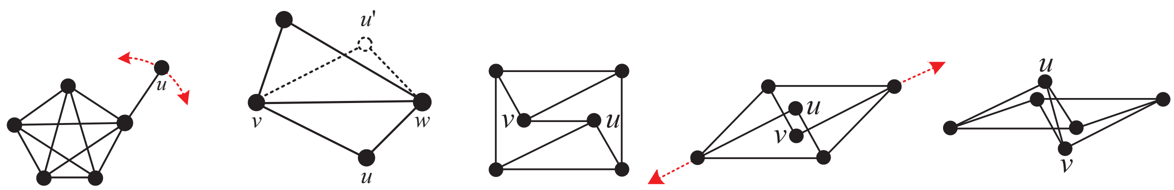
A network $\mathbb{N}$ can be modeled by a distance graph $G = \langle V, E, W \rangle$ where each vertex $v_i \in V$ denotes a node in the network and each edge $e \in E$ indicates the neighborhood of its two endpoint nodes. The measured distance between two neighboring nodes (including beacons) is defined by $W(e)$ (or $W(v_i, v_j)$) for $e = (v_i, v_j) \in E$.

B. Graph Rigidity and Realization

A *realization* of a graph G is a pair (G, p) where $G = \langle V, E, W \rangle$ and p is a function mapping the vertices in V to points in a Euclidean space (this study assumes 2-dimension space). Generally, realizations are referred to the *feasible* ones that respect the pairwise distance constraints between a pair of vertices u and v if the edge $(u,v) \in E$. That is to say, $\| \ p(u) - p(v) \ \|$ = $W(u,v)$ for all $(u,v) \in E$ in (G,p) where $\| \cdot \|$ denotes the Euclidean norm in $\mathbb{R}^d$. A distance graph G has at least one feasible realization which represents the ground truth of the corresponding network. Formally, G is embeddable in 2D space and all pairwise distances are compatible.

Two realizations (G,p) and (G,q) are *equivalent* if corresponding edges have the same lengths, that is, if $\| \ p(u) - p(v) \ \| = \| \ q(u) - q(v) \ \|$ holds for all pairs u, v with $(u,v) \in E$. Realizations (G,p), (G,q) are *congruent* if $\| \ p(u) - p(v) \ \| = \| \ q(u) - q(v) \ \|$ holds for all pairs u, v with $u, v \in V$. This is the same as saying that (G,q) can be obtained from (G,p) by an isometry of $\mathbb{R}^d$. We shall say that (G,p) is *globally rigid*, or that (G,p) is a *unique realization* of G, if every realization which is equivalent to (G,p) is congruent to (G,p)[3]. In other words, a graph is globally rigid if it is uniquely realizable [4]. A realization is generic if the vertex coordinates are algebraically independent. Since the set of generic realizations is dense in the realization space, almost all realizations are generic and we omit this word hereafter.

For a distance graph, there are several distinct manners in which the non-uniqueness of realization can appear. A graph that can be continuously deformed while still satisfying all the constraints is said to be *flexible*, as shown in Figure 1(a); otherwise it is *rigid*. Hence, rigidity is a necessary condition for global rigidity. Rigid graphs, however, are still susceptible to discontinuous



(a) Not rigid. (b) Not 3-connected. (c) Not redundantly rigid.

Figure 1. Realization non-uniqueness.

flex. Specially, they can be subject to flip (or fold) ambiguities in which a set of nodes have two possible configurations corresponding to a "reflection" across a set of mirror nodes (e.g., v and w in Figure 1(b)). This type of ambiguity is not possible in 3-connected graphs. A graph is said to be *3-connected* if there does not exist any set of two vertices whose removal disconnects the graph. Figure 1(c) further shows a 3-connected and rigid graph which becomes flexible upon removal of an edge. After the removal of the edge (u, v), a subgraph can swing into a different configuration in which the removed edge constraint is satisfied and then reinserted. This type of ambiguity is eliminated by *redundant rigidity*, the property that a graph remains rigid upon removal of any single edge.

Summarizing the above observations, Jackson and Jordan provides the necessary and sufficient condition for global rigidity in the following theorem.

Theorem 1 ([3])**.** *A graph with $n \geq 4$ vertices is globally rigid in 2 dimensions if and only if it is 3-connected and redundantly rigid.*

Based on Theorem 1, global rigidity can be tested in polynomial time by combining existing algorithms for rigidity [4], [5] and 3-connectivity [6]. As we know, a globally rigid graph can be uniquely determined if fixing any group of 3 vertices to avoid trivial variation in 2D plane, such as translation, rotation, or reflection. Hence, a network with at least 3 beacons is entirely and uniquely localizable if and only if its distance graph is globally rigid.

III. Overview

A. Outlier Distance Measurement

Given a distance graph $G = \langle V, E, W \rangle$, W is determined by the observed information, such as measured inter-node distances and positions of beacons. Because of the presence of noises, those measurements could be corrupted. Formally, for some edge $e = (u, v) \in E$, $W(e)$ is not necessarily identical with the ground truth distance between nodes u and v, i.e., $\| \pi(u) - \pi(v) \|$.

In general, there are two kinds of ranging errors in a localization system: *noisy error* and *outlier error*. Coming from environment factors and computation precision, noisy errors are moderate and predictable. Noisy errors can be alleviated by some optimization approaches like [7], [8], [9]. By contrast, outlier errors are much severer and more unpredictable, caused by hardware malfunction or failure, adversary attacks and etc. In this study, we focus on outlier errors, which is highly required to be eliminated for high accuracy localization.

Definition 1. Given a distance graph $G = \langle V, E, W \rangle$, an edge $e = (u, v) \in E$ is a normal edge if $W(e) = \| \pi(u) - \pi(v) \|$; otherwise, e is an outlier edge.

In this definition, we assume that no ranging noises are contained in normal edges, which proves to be an excellent starting point to characterize outlier measurements. In practice, we can introduce error tolerance and define a normal edge e as $|W(e) - \| \pi(u) - \pi(v) \| | < \epsilon$. The parameter ϵ is dynamically adjusted according to various ranging techniques and application requirements.

Note that outlier edge is defined based on the ground truth location π that is unknown. Under noisy measurements, even though localization can produce a feasible solution, the solution is one of the location estimations that fits the distance constrains well. The real locations of nodes are still not guaranteed. Without the ground truth information, we turn to the redundant information of G that can be used to tell whether an edge is outlier or not.

B. Redundant Information in G

When a node u obtains the distance between itself to one of its neighbors v, how can we know this measurement is correct or not? To verify $e = (u, v)$, we need to exploit the remaining graph $G - e$ after the removal of e. If $G - e$ provides enough correct information, it is possible to verify e. In particular, for a (satisfied) realization $(G - e, p)$ of $G - e$, the value of $\| p(u) - p(v) \|$ can be used to verify e.

The graph $G - e$ falls into 3 cases, as shown in Figure 2 where $G - (u, v)$ means the subgraph with edge $e = (u, v)$ deleted. In the first case, the removal of e results in a flexible graph, in which u and v can

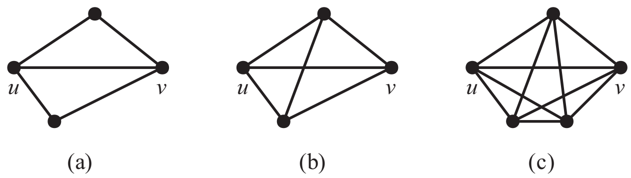


Figure 2. 3 cases of $G-(u,v)$. (a) $G-(u,v)$ is flexible; (b) $G-(u,v)$ is rigid; (c) $G-(u,v)$ is globally rigid.

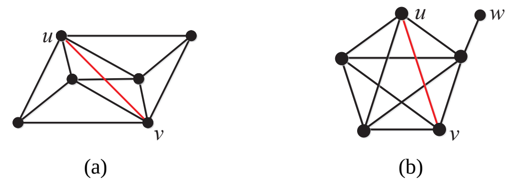


Figure 3. Two examples of verifiable edges. (a) G and $G-(u,v)$ are both globally rigid. (b) G and $G-(u,v)$ are not globally rigid while $G-(u,v)-w$ is a globally rigid component.

continuously deform. Accordingly, in any realization $(G-e,p)$, the value of $\| p(u)-p(v) \|$ should be within a number of continuous intervals. Taking Figure 2(a) as an example, $\| p(u)-p(v) \|$ must satisfy the triangle inequality of both two triangles. In the second case, $G-e$ remains rigid and in any realization $(G-e,p)$, $\| p(u)-p(v) \|$ has a series of discrete values since $(G-e,p)$ only allows discrete deformations. In Figure 2(b), u can flip over to the right side, yielding two possible values of $\| p(u)-p(v) \|$. In the last case, $G-e$ is globally rigid and $\| p(u)-p(v) \|$ has a unique value for all $(G-e,p)$. Such a unique value, comparing with $W(e)$, can be used as an important evidence of outlier existence or inexistence. To summarize, the first two cases indicate whether e is reasonable; i.e., $W(e)$ in a reasonable range. The third case guarantees the correctness (or incorrectness) of $W(e)$. This work focuses on the third case: $G-e$ is globally rigid; or more rigorously, $\| p(u)-p(v) \|$ has a unique value over all realization p of $G-e$.

We first assume that $G-e$ contains no outlier edge in the following analysis. The case that $G-e$ contains several outlier edges is discussed in Section V-D.

IV. Verifiable Edge

In this section, we analyze how to determine an edge to be normal or outlier without ground truth information. Applying graph rigidity, we define edge verifiability and study the conditions for an edge being verifiable.

A. Definition

For an edge $e=(u,v)$, if the inter-node distance measurements, rather than $W(e)$, are sufficient to figure out the distance between u and v, $W(e)$ is verifiable. We formulate the edge verifiability by utilizing the redundant information in G as follows:

Definition 2. Given a distance graph $G=\langle V,E,W\rangle$, an edge $e=(u,v)\in E$ is referred to be *verifiable* if $\| p(u)-p(v) \| = \| q(u)-q(v) \|$ holds for any two equivalent realizations $(G-e,p)$ and $(G-e,q)$ of $G-e$.

Considering an edge $e=(u,v)$ that is not verifiable, there exists at least two distinct values of $\| p(u)-p(v) \|$ for all $(G-e,p)$. Thus, the correctness of $W(e)$ cannot be judged with assurance.

B. Conditions for Edge Verifiability

This sub-section discusses the conditions of an edge being verifiable, which can be used to find verifiable edges in a graph.

Theorem 2. *In a graph $G=\langle V,E,W\rangle$, an edge $e=(u,v)\in E$ is verifiable in G if $G-e$ is globally rigid, or the two ends u and v are included in at least one globally rigid component of $G-e$.*

Proof: Let C denote one globally rigid component in $G-e$ that contains u and v. By definition, for any two equivalent realizations $(G-e,p)$ and $(G-e,q)$, $\| p(v_i)-p(v_j) \| = \| q(v_i)-q(v_j) \|$ holds for all $(v_i,v_j)\in E$. Since $C\subset G-e$, we have $\| p(v_i)-p(v_j) \| = \| q(v_i)-q(v_j) \|$ for all $(v_i,v_j)\in E(C)$. According to the global rigidity of C, the fact that $\| p(v_i)-p(v_j) \| = \| q(v_i)-q(v_j) \|$ for all $(v_i,v_j)\in E(C)$ suggests $\| p(u)-p(v) \| = \| q(u)-q(v) \|$, where p and q are any equivalent realizations of $G-e$. Thus, $e=(u,v)$ is verifiable by Definition 2. ■

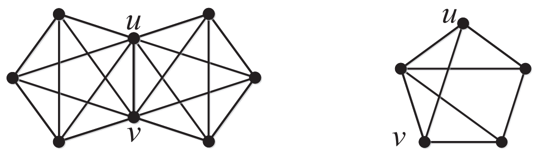
As illustrated in Figure 3(a), edge (u,v) colored red is verifiable since $G-(u,v)$ is globally rigid. The fact that an edge $e=(u,v)$ is verifiable does not require $G-e$, even G, being globally rigid. The only thing we need is the existence of a globally rigid component containing u and v, when $G-e$ is not globally rigid. Figure 3(b) demonstrates an example of such situations. In this case, $G-(u,v)$ is not globally rigid, but (u,v) is verifiable because u and v are both contained in a globally rigid component $G-(u,v)-w$.

V. Verifiable Graphs

We expand the concept of verifiability from edges to graphs. Verifiable graphs have a nice property of outlier resistance. In this section, we explore the conditions of a graph being verifiable.

A. Definition

Generally, verification should be carried out not only for several specific edges, but also the entire graph. This leads us to the problem of graph verifiability, in which



(a) $\delta(G) = \frac{n}{2}$. (b) $\delta(G) = \frac{n+1}{2}$.

Figure 4. Relationships between degree and global rigidity. (a) G is not globally rigid as suffering from a flip ambiguity by reflecting along line of (u, v). (b) The remaining graph of G is not globally rigid upon removing any edge.

all edges in a graph must be verified except the known accurate ones such as the edges connecting two beacons.

Definition 3. Given a distance graph $G = \langle V, E, W \rangle$, G is verifiable if e is verifiable for all $e \in E$.

Based on Theorem 2, the endpoints of a verifiable edge $e \in E$ are contained in a globally rigid component in $G - e$. We assume G is globally rigid in this section; otherwise G is not entirely localizable and measurement verification for those non-localizable nodes is less important. If G is globally rigid, we have the following theorem.

Theorem 3. *Given a globally rigid graph $G = \langle V, E, W \rangle$, G is verifiable if and only if $G - e$ is globally rigid for all $e \in E$.*

Proof: Since $G - e$ is globally rigid for all $e \in E$, all edges in G are verifiable and, by Definition 3, G is verifiable. For the necessity, we suppose the contrary that $G - e$ is not globally rigid for some edge e. The fact that e is verifiable implies that the endpoints of e must reside in one globally rigid component of $G - e$. Hence, adding e back, G is not globally rigid, a contradiction. ∎

B. Conditions for Graph Verifiability

To check whether a graph is verifiable, the straightforward method is to judge edge verifiability for all edges in the graph. In the following, we study the conditions of graph verifiability through general graph properties.

Jackson et al. have presented two sufficient conditions for global rigidity in terms of network connectivity and vertex degree of G in [10] and [3], respectively.

Lemma 1 ([10]). *Let $G = (V, E)$ be a 6-mixed-connected graph. Then $G - e$ is globally rigid for all $e \in E$.*

Lemma 2 ([3]). *Let $G = (V, E)$ be a graph on $n \geq 4$ vertices with $\delta(G) \geq \frac{n+1}{2}$, where $\delta(G)$ denotes the minimum degree in G. Then G is globally rigid.*

In Lemma 1, mixed vertex and edge connectivity is used. Let $G = (V, E)$ be a graph. A pair (U, D) with $U \subset V$ and $D \subset E$ is a mixed cut in G if $G - U - D$ is not connected. We say that G is *6-mixed-connected* if $2|U| + |D| \geq 6$ for all mixed cuts (U, D) in G. Equivalently, G is 6-mixed-connected if G is 6-edge-connected, $G - v$ is 4-edge-connected for all $v \in V$, and $G - \{u, v\}$ is 2-edge-connected for all pairs $u, v \in V$. We also know that 6-vertex-connectivity is sufficient for 6-mixed-connectivity. We formulate above discussion in the following corollaries.

Corollary 1. *If a graph G is 6-vertex-connected or 6-mixed-connected, G is verifiable.*

The conclusion on vertex connectivity is best possible and cannot be reduced from six to five. The examples of 5-vertex-connected graphs being not globally rigid are given in [11].

Corollary 2. *For a graph $G = \langle V, E, W \rangle$ on $n > 4$ vertices, if $\delta(G) \geq \frac{n+3}{2}$ where $\delta(G)$ denotes the minimum degree in G, G is verifiable.*

The lower bound on the minimum degree in Lemma 2 is best possible, as shown in Figure 4(a) by two complete graphs of equal size with two vertices in common. It is easily deduced from condition of $\delta(G) \geq \frac{n+3}{2}$ that $\delta(G - e) \geq \frac{n+1}{2}$ for any $e \in E$, which indicates $G - e$ is globally rigid for all $e \in E$ by Lemma 2. Thus G is verifiable. In the following analysis, we provide a more accurate conclusion. For a graph on n vertices, if $n \leq 4$, even the complete graph is not verifiable. If $n = 5$, Figure 4(b) shows the minimum degree of 4 is required. Other cases ($n > 5$) are complicated and discussed in the following lemma.

Lemma 3. *For a graph $G = \langle V, E, W \rangle$ on $n > 5$ vertices, if $\delta(G) \geq \frac{n+1}{2}$ where $\delta(G)$ denotes the minimum degree in G, $G - e$ is globally rigid for all $e \in E$.*

Proof: According to Lemma 2, G is globally rigid as $\delta(G) \geq \frac{n+1}{2}$. Since $\delta(G) \geq \frac{n+1}{2}$, we have $\delta(G-e) \geq \frac{n-1}{2}$ and all but at most two non-adjacent vertices have degree at least $\frac{n+1}{2}$. To prove $G - e$ is globally rigid, it suffices to show $G - e$ is 3-connected and redundantly rigid for all $e = (u, v) \in E$.

For a contrary suppose that $G - e$ has a vertex cut of size less than 3. This vertex cut, denoted by C and $|C| \leq 2$, divides $G - e$ into a number of overlapped components, among which the one with minimum number of vertices is denoted by H and $3 \leq |H| \leq \frac{n}{2} + 1$. H contains at most one of u and v; otherwise C is still a cut in G and G is not globally rigid. We know that the complete graph of $|V|$ vertices has degree at most $|V| - 1$. If $|H| \leq \frac{n-1}{2}$ or $|H| = 3$, $\delta(G - e) \geq \frac{n-1}{2}$ implies at least one edge connecting a vertex in H (but not in C) and a vertex outside H.

Otherwise $\frac{n-1}{2} < |H| \leq \frac{n}{2} + 1$ and $|H| \geq 4$, since at most one vertex having degree $\frac{n-1}{2}$ and others at least $\frac{n+1}{2}$ in H, we know that at least one edge connecting a vertex in H (but not in C) and a vertex outside H. Both cases contradicts that C is a vertex cut. Hence, $G - e$ is 3-connected.

In the following we will show $G - e$ is redundantly rigid. Since G is globally rigid, $G - e$ is rigid. We suppose to the contrary that $H = G - e - f$ is not rigid for some edge $f = (s, t) \in E - e$. There are two cases of $\delta(H)$:

1) $\delta(H) \geq \frac{n-3}{2}$.
   In this case, one vertex has a degree of $\frac{n-3}{2}$; two vertices have a degree of $\frac{n-1}{2}$; others $\frac{n+1}{2}$.
2) $\delta(H) \geq \frac{n-1}{2}$.
   In this case, four vertices have a degree of $\frac{n-1}{2}$; others $\frac{n+1}{2}$.

Let C be a rigid component of H with minimum vertices. We know that C contains at most one of s and t. Since distinct rigid components share at most one vertex and $\delta(H)$, we have $|V(C)| \leq \frac{n-1}{2}$ and $|V(D)| \geq 4$. Since C is a rigid component of H, each vertex of D is adjacent to at most one vertex of C in H. Hence, there are two cases of $\delta(D)$:

1) $\delta(D) \geq \frac{n-5}{2}$.
   In this case, one vertex has a degree of $\frac{n-5}{2}$; two vertices have a degree of $\frac{n-3}{2}$; others $\frac{n-1}{2}$.
2) $\delta(D) \geq \frac{n-3}{2}$.
   In this case, four vertices have a degree of $\frac{n-3}{2}$; others $\frac{n-5}{2}$.

In both cases, we can construct a graph $\bar{D}$ by adding two edges to D, such that $\delta(\bar{D}) \geq \frac{n-1}{2}$. Since $|V(C)| \geq 2$, we have $|V(D)| \leq n - 2$. We may now use induction on n to deduce that $\bar{D} - g$ is globally rigid for any edge $g \in \bar{D}$. Hence, since $|V(D)| \geq 4$, $\bar{D} - g$ is redundantly rigid suggests D is rigid.

Considering $|V(C)|$ and $\delta(H)$, we have 3 cases:

1) If $|V(C)| = 2$, at least one vertex in C has a degree of 3 and is adjacent to two vertices in D.
2) If $|V(C)| = 3$, at least one vertex in C is adjacent to two vertices in D or all 3 vertices in C are adjacent to at least one vertex in D.
3) If $|V(C)| \geq 4$, since $|V(C)| \leq \frac{n-1}{2}$ and at least one vertex in C has a degree at least $\frac{n+1}{2}$, this vertex is adjacent to two vertices in D.

In all cases, H is rigid, a contradiction. ■

According to Lemma 3, we formulate the requirements on degree for verifiable graphs as follows.

Theorem 4. *For a graph $G = \langle V, E, W \rangle$ on $n > 5$ vertices, if $\delta(G) \geq \frac{n+1}{2}$ where $\delta(G)$ denotes the minimum degree in G, G is verifiable.*

C. Easily Verifiable Graphs

The verification of edges and graphs relies on graph realization. It is usually prerequisite to localize $G - e$ to achieve the value of $\parallel p(u) - p(v) \parallel$ to verity e. However, network localization is an NP-hard problem in general cases even when networks are localizable. Consequently, a large portion of verifiable graphs are not easily verifiable. Further explained, an edge e is verifiable does not mean it can be verified easily and efficiently in practice because calculating $\parallel p(u) - p(v) \parallel$ (or equivalently, localizing $G - e$) is usually computationally infeasible. Only when $G-e$ is an easily localizable graph such as trilateration graph (TRI) can edge e be easily verified.

The basic principle of trilateration is that the position of an object can be uniquely determined by measuring the distances to three reference positions. It is widely-used in many real-world applications[2], [12] as it is computationally efficient, fully distributed, and easy to implement. Importantly, the networks that can be constructed by iterative trilateration are localizable.

Theoretically, a trilateration ordering of a graph $G = (V, E)$ is an ordering $(v_1, v_2, \ldots, v_n)$ of V, in which the first 3 vertices are pairwise connected and at least 3 edges connect each vertex v_j , $4 \leq j \leq n$, to the set of the first $j - 1$ vertices. A graph is a trilateration extension if it has a trilateration ordering. It is shown that trilateration extensions are globally rigid[13], [14]. A large body of localization approaches are based on trilateration.

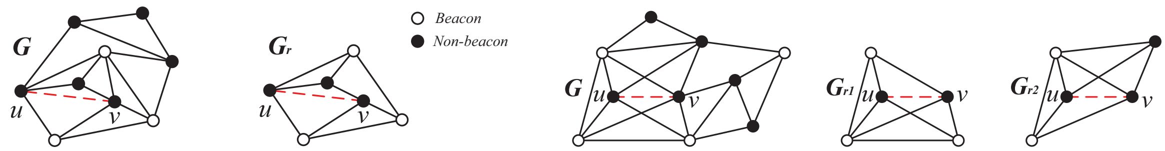
A graph is easily verifiable only if every edge is easily verifiable.

Definition 4. Graph $G = \langle V, E, W \rangle$ is easily verifiable if $G - e$ is easily localizable for all $e \in E$.

Similar to trilateration, we define quadrilateration graph as follows. A quadrilateration graph $G = (V, E)$ has a quadrilateration ordering $(v_1, v_2, \ldots, v_n)$ of V, in which the first 4 vertices are pairwise connected and at least 4 edges connect each vertex v_j , $5 \leq j \leq n$, to the set of the first $j - 1$ vertices. Using quadrilateration graphs, we have the following theorem:

Theorem 5. *If G is a quadrilateration graph, G is easily verifiable.*

Theorem 5 holds since $G - e$ is a trilateration graph if G is a quadrilateration graph. Note the fact that G is a trilateration graph does not imply G is easily verifiable (even not verifiable) because $G - e$ may not be 3-connected. Verification of easily verifiable graphs is practical; however, for those graphs that are not easily localizable, it remains a difficult and challengeable task of their verification.



(a) MGRC 1. (b) MGRC 2.

Figure 5. Minimally globally rigid component. G is the grounded graph where the dashed red edge is to be verified. G_r indicates a MGRC.

D. Minimally Globally Rigid Component

Recall the assumption in Section III-B that the graph $G-e$ provides enough and correct information to verify e. The proposed conditions solve the enough problem. The correctness, however, is not guaranteed since outlier edges may also be contained in $G-e$. Consequently, how to wisely avoid incorrect information in $G-e$ is a significant issue lying ahead.

To deal with this problem, we try to include as least (incorrect) information as possible in the verification procedure by using the minimum number of measurements in $G-e$. To achieve this purpose, we introduce the concept of *minimally globally rigid component(MGRC)*. A graph $G = (V, E)$ is minimally globally rigid if G is globally rigid, and no proper subgraph of G is globally rigid. A MGRC is a minimally globally rigid subgraph of G. According to the definition, the MGRCs containing u and v are capable of verifying edge $e = (u, v)$; at the meanwhile, involve the minimal amount of measurements. This fact implies that MGRC, to the large extent, limits the existence of outlier measurements. As illustrated in Figure 5, each G_r indicates such a MGRC of $G-e$ for verifying edge $e = (u, v) \in E$ in G. During the procedure of edge verification, we follow the *MGRC Rule* which means to verify each edge using one of its MGRCs. MGRC Rule increases the accuracy of verification and meanwhile reduces the costs since the problem scale is decreased. However, two key challenges exist to implement the MGRC Rule: First, how to find a MGRC containing both ends of the edge to be verified; Second, such MGRCs may not be unique.

With respect to the nonuniqueness, we prefer the MGRC of the fewest edges among all possible MGRCs. For the special case that all possible MGRCs have the same number of edges (e.g., G_{r1} and G_{r2} in Figure 5(b)), we treat beacons more credible than other nodes. The reason is that locations of beacons are more trusted and the distance measurements from ordinary nodes to beacons are accordingly more reliable. As illustrated in Figure 5(b), G_{r1} and G_{r2} are two MGRCs with the same number of vertices and edges. We prior choose G_{r1} to verify (u, v) as it contains one more beacon than G_{r2}.

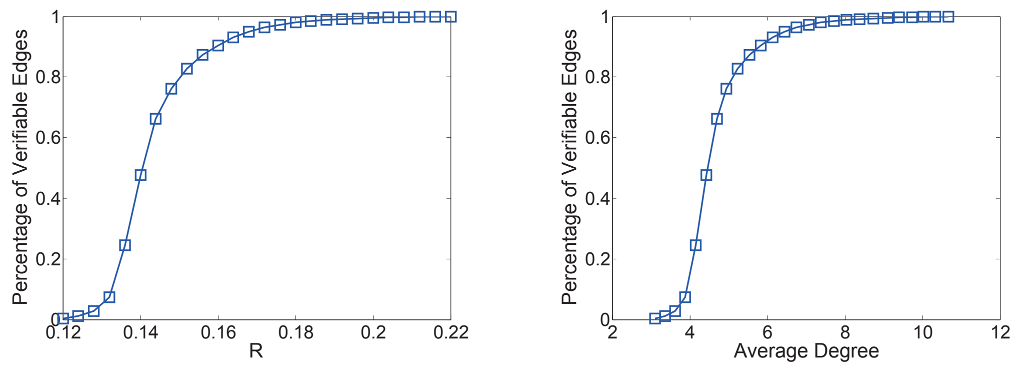
The other issue, finding a MGRC, is complicated. To the best of our knowledge, there are no existing theory and methods for finding a MGRC yet. In this work, we tentatively take a globally rigid component that contains relatively fewer edges instead of the optimal one, which also results in a melioration of avoiding using outlier edges during verification. It is a good direction of future research to establish theoretical foundations and design algorithms for MGRC discovery.

VI. Evaluation

We evaluate the performance of the proposed edge verifying and graph verifying approaches through extensive simulations. We randomly generate networks of 100 nodes which are uniformly distributed in a unit square $[0, 1]^2$. The disk model with a radius is adopted for distance measurement. The communication range is denoted by R. For each network instance, we change the value of R from 0.12 to 0.22 stage-by-stage with a step length of 0.04. For each setting, we integrate results from 100 network instances.

Figure 6 plots the percentage of verifiable edges in different node densities. A simple trend in Figure 6(a) is that more edges are verifiable in dense networks, since sufficient distance constraints are available. For ease of understanding the relationship between average degree and percentage of verifiable edges, we calculate the average degree of 100 network instances in different settings of R, and plot Figure 6(b). We find that only a few edges are verifiable when the average degree is below 4, but the percentage of verifiable edges increases sharply to over 90% as the average degree grows from 4 to 6. All edges are verifiable when the average degree is about 10.

To examine the relationship between node connectivity and graph verifiability, we change the value of R from 0.17 to 0.26 and re-run the simulations. The results are plotted in Figure 7. When the network is sparse (R <0.17), no instance is verifiable. The percentage of verifiable graphs increases near-linearly as R grows from 0.18 to 0.24. Simulation results also show that all instances are verifiable only when the networks are dense. In Figure 7(b), all graphs are verifiable when the average degree is over 14. The degree of 14 is much lower than the degree constraint of $\frac{n+1}{2}$ in Theorem 4, which should approximately equal 51 in



(a) Percentage of verifiable edges v.s. R

(b) Percentage of verifiable edges v.s. Average Degree

Figure 6. Percentage of verifiable edges in different node densities.

this network deployment. This result demonstrates the efficiency and feasibility of proposed graph verifying method in practice.

Edge verification is an intermediate processing procedure between inter-node distance measurement and range-based localization. Our findings can be used to sift outlier ranging information and improve the accuracy of localization. To further show the gain of outlier sifting, we use multilateration as the basic localization method, and compare the localization performance of the two strategies of outlier sifting and no sifting. Note that our outlier detection approach works well with arbitrary localization methods. Here we employ multilateration just because it is a widely used localization technique.

We randomly generate 100 network instances of 100 nodes which are uniformly distributed in a unit square $[0, 100]^2$. The distance measurement range of each node is about 28, and all instances have a trilateration ordering. We choose these network parameters to ensure every node can be localized by trilateration, or equivalently, the graph is easily verifiable. Two parameters which influence the localization performance are the percentage of outliers in the graph (denoted by a) and the error of outliers (denoted by b). Let D_1 denote measured distance, D_2 denote the actual distance, we use $\frac{|D_1 - D_2|}{D_2}$ to calculate the value of b. We set a=0.1, b=0.5 as the basic scenario, and change the value of a and b respectively to run the simulation. For edges other than outliers, we add a random ranging noise which is less than 1% of the actual distance to them. Average location error is used as the localization performance metric.

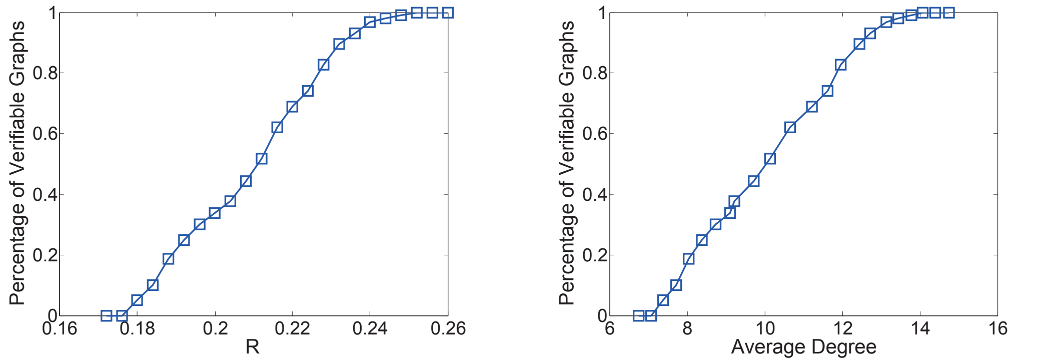
The results of 100 network instances are integrated and plotted in Figure 8. Without sifting, average location error tends to increase as the values of a or b increase. In contrast, the curves labeled Sifting keep almost unchanged, which means the location error is almost unacted on the influence of outlier measurements as most of them are correctly sifted. In addition, the average location error with sifting is much smaller than without sifting. The gap between these two cases reaches up to 40% in the basic scenario with a=0.1 and b=0.5. All these results show the effectiveness of outlier sifting through edge verification.

VII. Related Work

A. Localization Literature

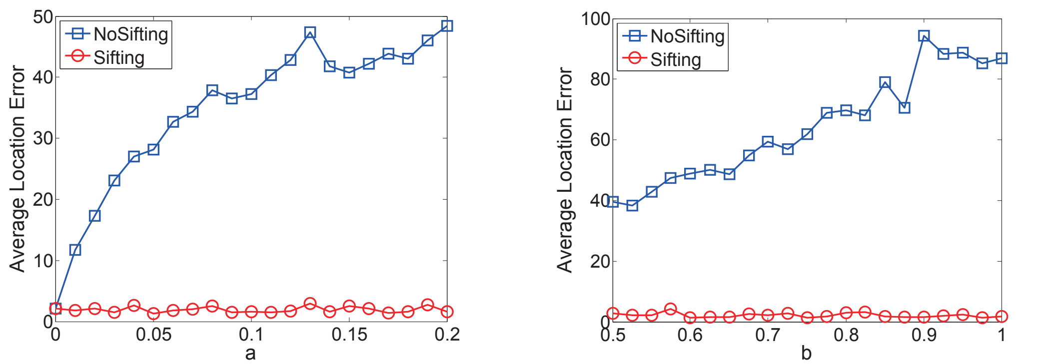
Localization is essential for many environment monitoring or surveillance applications[17], [18]. Existing solutions fall into two categories. Range-based approaches assume nodes are able to measure inter-node distances; while range-free ones merely use neighborhood information. Many localization algorithms are range-based [19], [12], [2], [20], adopting distance ranging techniques, such as Radio Signal Strength (RSS) [1] and Time Difference of Arrival (TDoA) [12]. RSS maps received signal strength to distance according to a signal attenuation model, while TDoA measures the signal propagation time for distance calculation. In practice, RSS-based ranging measurements contain noise on the order of several meters [19], especially in rigorous environments. On the contrast, TDoA is impressively accurate and obtains close to centimeter accuracy for node separations under several meters in indoor environments [2], [21]. Due to the hardware limitations and energy constraints of wireless communication devices, range-free approaches are cost-effective alternatives. Most existing range-free approaches compute node locations according to network connectivity measurements[28], [29].

The majority of localization algorithms [19], [22], [12], [2] assume a dense network such that iterative trilateration (or multilateration) can be conducted. Other methods [23] record all possible locations in each positioning step and prune incompatible ones whenever



(a) Percentage of verifiable graphs v.s. R (b) Percentage of verifiable graphs v.s. Average Degree

Figure 7. Percentage of verifiable graphs in different node densities.



(a) b=0.5 (b) a=0.1

Figure 8. Benefit of sifting outliers.

possible, which, in the worst case, can result in an exponential space requirement. Besides, some works [13], [24], [25] study the relationship between network localization and rigidity properties of ground truth graphs.

B. Graph Rigidity Literature

Graph rigidity has been well studied in mathematics and structural engineering [4], [15], [21], having a surprisingly large number of applications in many areas. In rigidity literature, many efforts have been made to explore the combinatorial conditions for rigidity. Laman [15] first pointed out that a graph $G = (V, E)$ is generically rigid if it has a induced subgraph in which edges are "independently" distributed. The statement also leads to an $O(|V|^2)$ algorithm [5] for rigidity test. For global rigidity, a sufficient and necessary condition [3] is presented based on the results in [4] by combining both redundant rigidity and 3-connectivity. Recently, Jackson and Jordan [10] prove a sufficient condition of 6 mixed connectivity, which improves a previous result of 6-connectivity by [11].

C. Error Control Literature

Because of the inevitability of noise in practice, how to control error accumulation in localization also attracts a lot of research efforts. Error management[7] uses error registries to select nodes that participate in the localization procedure based on their relative contribution to the localization accuracy, and [8] requires "robust quadrilaterals" to prevent large location errors introduced by flip ambiguities. From the perspective of security, outlier-resistant localization has also been studied. Minimum Mean Square Estimation (MMSE) is used to identify and remove malicious nodes in [8], and Least Median of Squares (LMS), an estimator with high breakdown point, is adopted in [26]. A speculative filtering algorithm is designed in [9] to detect phantom nodes. Deriving inspiration from robust statistics, the recent work SISR [27] uses a residual shaping influence function to de-emphasize the "bad nodes" and "bad links" during the localization procedure. Besides requiring dense links, SISR is a centralized algorithm, which may prevent it from applying to large-scale deployments.

VIII. Conclusion

To eliminate the negative impact of inaccurate measuremnts, outlier detection serves as an essential and prior component for all range-based localization approaches. In this study, we explore how to identify outliers by using redundant distance information. Applying graph rigidity theory, we establish the theoretical foundation of outlier detection problem and analyze the conditions of a specific measurement being an outlier. The evaluation results show that the proposed method improves the location accuracy by modestly and wisely rejecting outliers during localization.

A direction of future research with good potential is to further exploit the redundant information in a graph, especially when $G - e$ is not rigid. In this case, the constrains of e are more diversified and complicated.

Acknowledgment

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